Assignment 3

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Sampling, Estimating, Testing

Instructions: You have time until 22 January 2021, 23:55 h, to submit your solution to this assignment on OLE. This means, you have 11 days time to submit your answers. Recall that your answers to each question count towards the final mark. Therefore, it is worthwhile to submit the assignment even if you feel not able to answer all questions.

- You can work out your solutions with a word processing system (Word, Latex) or by hand. It may be easier to write up your answers by hand because they are likely to contain symbolic calculations with fractions, powers, and integrals. If you submit handwritten solutions, make an effort to write clearly and to structure and comment your answers so that they are legible. If you write by hand, submit the answer as a scanned PDF document. (Don't submit a photo, but a scan. Photos are usually very hard to read.)

- Also if you write by hand, organize your submission clearly and and write legibly. Otherwise, we cannot accept your work.

- Explain your answers and the approach by which you obtained them. Mention when you apply specific rules (like "the variance of a sum of independent variables is the sum of the variances"). A chain of formulas alone is not enough.

- Each question has a weight, expressed in points. For your answer you get a mark on the scale from 0 to 30. The weight determines how much the mark contributes to the coursework mark and therefore to the final mark. Note that for the final mark, we consider the maximum of the exam mark and the mark for the question. Therefore, not much is lost if you get a low mark for one question: (i) it can be compensated by the exam mark, and (ii) it does not influence the marks for the other questions.

- Your assignment should represent your own effort. However, you are not expected to work alone. It is fine to discuss the exercises and try to find solutions together, but each student shall write down and submit his/her solutions separately.

- If you are a student who took this course already last year, you need not submit the same answers as past year to get points. Only submit answers to a question if they are different from the answers of last year. For other questions just indicate in your submssion that you would like to get the same mark as before. That spares us repeatedly marking the same work.

1 Lifetime of Bulbs

The lifetime (in hours) of a type of electric bulb has expected value 5,000 and standard deviation 800. Approximate the probability that the sample mean of n such bulbs is greater than 5,250 when

1. $n = 36$;
2. $n = 49$;
3. $n = 64$.

(Weight: 16% of this CW)

2 Nicotine in Cigarettes

A sample of 20 cigarettes is tested to determine nicotine content and the average value observed was 1.2mg.

1. Compute a 99 percent two-sided confidence interval for the mean nicotine content of a cigarette if it is known that the standard deviation of a cigarette's nicotine content is $\sigma = .2$mg.
2. In the above problem, suppose that the population variance is not known in advance of the experiment. If the sample variance is $.04$, compute a 99 percent two-sided confidence interval for the mean nicotine content.
3. Under the assumptions of Item 2, compute a value c for which we can assert "with 99 percent confidence" that c is larger than the mean nicotine content of a cigarette. Choose a value c as small as possible.

(Weight: 20% of this CW)

3 Debt of Credit Card Accounts

A random sample of 300 credit cardholder accounts indicated a sample mean debt of \$6,120 with a sample standard deviation of \$4,200.

1. Construct a 95 percent confidence interval estimate of the average debt of all cardholders.
2. Find the smallest value c that "with 90 percent confidence," exceeds the average debt per cardholder.

(Weight: 20% of this CW)

4 Weight of Beryllium

The amount of beryllium in a substance is often determined by the use of a photometric filtration method. If the weight of the beryllium is μ, then the value given by the photometric filtration method is normally distributed with mean μ and standard deviation σ. A total of eight independent measurements of 3.180 mg of beryllium gave the following results:

$$3.166,\ 3.192,\ 3.175,\ 3.180,\ 3.182,\ 3.171,\ 3.184,\ 3.177$$

Use the preceding data to

1. estimate μ;
2. find a 99 percent confidence interval estimate of μ.

(Weight: 16% of this CW)

5 Random Mice

A colony of laboratory mice consists of several thousand mice. The average weight of all the mice is 32 grams with a standard deviation of 4 grams. A laboratory assistant was asked by a scientist to select 25 mice for an experiment. However, before performing the experiment the scientist decided to weigh the mice as an indicator of whether the assistant's selection constituted a random sample or whether it was made with some unconscious bias (perhaps the mice selected were the ones that were slowest in avoiding the assistant, which might indicate some inferiority about this group).

- If the sample mean of the 25 mice was 30.4, would this be significant evidence, at the 5 percent level of significance, against the hypothesis that the selection constituted a random sample?

(Weight: 12% of this CW)

6 Measuring the pH Value

In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20. A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of .02. Suppose 10 independent measurements yielded the following pH values:

$$8.18,\ 8.17,\ 8.16,\ 8.15,\ 8.17,\ 8.21,\ 8.22,\ 8.16,\ 8.19,\ 8.18$$

1. What conclusion can be drawn at the $\alpha = .10$ level of significance?
2. What about at the $\alpha = .05$ level of significance?
3. What is the p-value of the test?

(Weight: 16% of this CW)