9 Estimation, Confidence Intervals, and p-Values

9.1 Probability Estimation Based on Parameter Estimation

A manufacturer of heat exchangers requires that the plate spacings of its exchangers be between .240 and .260 cm. A quality control engineer sampled 20 exchangers and measured the spacing of the plates on each exchanger. If the sample mean and sample standard deviation of these 20 measurements are .254 cm and .005 cm, estimate the fraction of all exchangers whose plate spacings fall outside the specified region. Assume that the plate spacings have a normal distribution.

Hint: Take sample mean and sample variance as estimates for the real mean and variance and derive an estimate

9.2 P-values of a Population Mean

A population distribution has standard deviation 15. Find the p-value of a test of the hypothesis that the population mean is 50 if the average of 64 observations is

1. 52

2. 54

3. 56

How does the assumption that the population distribution is normal or not influence your answer?

9.3 Confidence Intervals for Measures

A measuring device returns an answer with an error that is normally distributed with mean 0 and standard deviation 0.2. Suppose that one object is measured 5 times, and the device returns the values

$$3.152, \quad 3.139, \quad 3.162, \quad 3.147, \quad 3.144.$$

1. Determine the 95% and 99% confidence interval estimates for the true measure.

2. Compute the p-value for the hypothesis $H_0 : \mu = 3.0$.

3. Compute the p-value for the hypothesis $H_0 : \mu \leq 3.0$.

4. How would you change your answers if the standard deviation were unknown?

Hint: Use R to compute the sample mean and variance.

1. Define a vector consisting of the height measurements:

   ```
   MEAS = c(3.152, 3.139, 3.162, 3.147, 3.144)
   ```

   The function `c(.)` is read "concatenate" and concatenates is input to a vector.

2. Apply the functions `mean(.)` and `var(.)` to get the sample mean and sample variance of the vector:

   ```
   smean = mean(MEAS)
   svar = var(MEAS)
   ```

   To get the sample standard deviation, take the square root of the sample variance by means of the function `sqrt(.)`.

9.4 Salaries of Politicians

A random sample of 36 politicians shows that their mean annual salary is €215,500 with a sample standard deviation of €11,000.

1. Determine the 95% two-sided and lower and upper one-sided confidence interval for the mean salary of all politicians.

2. Compute the p-value for the hypothesis $H_0 : \mu = 210,000$

3. Compute the p-value for the hypothesis $H_0 : \mu \leq 210,000$