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8 Normal Distribution and Central Limit Theorem

Solve these exercises first using a standard normal table to compute the values (you can find one on OLE). Then, once you are done, you can use RStudio to compute the values instead.

8.1 Snowfall in Bolzano

The annual snowfall in Bolzano is normally distributed with mean 29cm and standard deviation 3.6cm (recall that the standard deviation is the square root of the variance).

(i) What is the probability of having more than 33cm of snow next year?

(ii) What is the probability that at least 66cm of snow fall over the next two years?

(iii) What is the probability that at least 99cm of snow fall over the next three years?

(iv) Did you make any independence assumptions for the last two items?

8.2 Sum of Points of Dice

If 50 fair dice are rolled, approximate the probability that the sum of the values obtained (which ranges from 50 to 300) is between 150 and 200 inclusive.

8.3 Drug Testing

A clinical experiment for a new drug is organized and 25 men and 35 women volunteer to take part in a clinical experiment for the drug. The results of the experiments are independent, and the drug has a probability of success of 50% on men, and of 75% on women. Let $\mathcal{X}$ be the number of volunteers on which the drug was successful, and $\mathcal{X}_M$, $\mathcal{X}_W$ the number of men and women in the experiment where the drug was successful.

1. Is $\mathcal{X}$ a binomial random variable?
2. What are the distributions of $\mathcal{X}_M$ and $\mathcal{X}_W$?
3. What is the relationship between the three random variables?

4. Approximate the probability that $\mathcal{X} > 35$.

8.4 Accuracy of Distance Measurement

An astronomer wants to measure the distance from earth to a distant star. However, due to atmospheric disturbances, any measurement will yield the distance d plus or minus some error. As a result, the astronomer has decided to make a series of measurements and then use their average value as an estimate of the actual distance. The astronomer believes that the values of the successive measurements are independent random variables with a mean of d light years and a standard deviation of 2 light years.
How many measurements need he make to be at least 95 percent certain that his estimate is accurate to within $\pm.5$ light years?