

7 Random Variables

7.1 Erroneous Bits

A communication channel transmits digits 0 and 1. The digit transmitted has a probability 0.2 of being wrongly received. To reduce the likelihood of error, every bit is tripled before transmission (that is, 0 is sent as 000 and 1 is sent as 111), and the receiver interprets the original digit through a majority rule. If a 2-digit message is sent, what is the probability of it being incorrectly decoded? Which independence assumptions did you make?

7.2 A Binomial Random Variable

Let \mathcal{X} be a binomial random variable with $E[\mathcal{X}] = 9$ and $Var(\mathcal{X}) = 2.25$. Find:

1. $P[\mathcal{X} = 7]$
2. $P[\mathcal{X} > 13]$

7.3 Winning in the Lottery

If you buy a lottery ticket for 40 different lotteries, and each of them gives you a chance of winning of $1/100$, what is the approximate probability that you will win in

- (i) at least one,
- (ii) exactly one, and
- (iii) at least two lotteries?

Hints: What kind of distribution is this? Remember that you can either compute the distribution as it is, or that alternatively you can approximate that distribution by another one.

As a third possibility, install RStudio on your computer and calculate the exact probability.

7.4 Hamburgers Sold by a Restaurant

A restaurant sells, in average, 121.95 hamburgers every week. Give an estimate for:

- (i) the proportion of weeks where more than 130 hamburgers were sold; and
- (ii) the proportion of weeks selling at most 100 hamburgers.

Explain your answer. It suffices to give the answer as algebraic expressions.

Hints: What kind of distribution is this?