

6 Variance and Estimates with Inequalities

6.1 Calculate the Variance

Suppose that \mathcal{X} has the density function

$$f(x) = \begin{cases} x^2(2x + 3/2) & 0 < x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Consider now the variable $\mathcal{Y} = \frac{2}{\mathcal{X}} + 3$. Compute the variance of \mathcal{Y} .

Hint: Think which property of variance may be helpful in determining the variance of \mathcal{Y} .

6.2 Student Test Scores

From past experience, a professor knows that the test score of a student taking her final examination is a random variable with mean 75.

1. Give an upper bound to the probability that a student's test score will exceed 85.
2. Suppose in addition the professor knows that the variance of a student's test score is equal to 25. Does that help to improve the bound?
3. What can be said about the probability that a student will score between 65 and 85?
4. How many students would have to take the examination so as to ensure, with probability at least 0.9, that the class average would be within 5 of 75?