Lab 5

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5 Joint Distributions and Expected Values

5.1 Joint Distributions and Independence

Suppose that the random variables $\mathcal{X}$, $\mathcal{Y}$ have the joint density function

$$f(x,y) = \begin{cases} c \cdot x \cdot y & 0 \leq x \leq 1,\, 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

1. Compute the constant c.
2. Compute the marginal densities $f_\mathcal{X}$, $f_\mathcal{Y}$ of $\mathcal{X}$ and $\mathcal{Y}$ respectively.
3. Are $\mathcal{X}$ and $\mathcal{Y}$ independent?

Now consider two different random variables. Suppose that $\mathcal{X}$, $\mathcal{Y}$ have the joint density function

$$g(x,y) = \begin{cases} c \cdot x \cdot y & 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find out in which way the two settings are different: What are the shapes of the areas where $f > 0$ and where $g > 0$?

4. Compute the constant c.
5. Compute the marginal densities $g_\mathcal{X}$, $g_\mathcal{Y}$ of $\mathcal{X}$ and $\mathcal{Y}$ respectively.
6. Are $\mathcal{X}$ and $\mathcal{Y}$ independent?

5.2 Cost of a Phone Repair

The time it takes to repair a mobile phone is a random variable whose density, in hours, is given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

The cost of the repair depends on the time it takes and is equal to $40 + 30\sqrt{x}$ when the time is x. Compute the expected cost to repair a mobile phone.

5.3 Law Of The Unconscious Statistician (LOTUS)

Suppose that $\mathcal{X}$ has density function

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Compute the expected value $E[\mathcal{X}^n]$ in two alternative ways:

1. by computing the density of $\mathcal{X}^n$ and then using the definition of expectation;
2. by using Proposition 39 (the "LOTUS Theorem"[1])

[1] LOTUS is short for "Law Of The Unconscious Statistician".