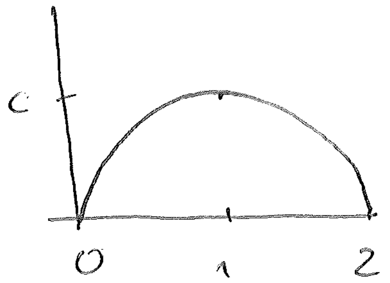
Example 28

$$f(x) = \begin{cases} c(2x - x^2) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

f(x):



Which c turns f into a density?

$$1 = c \int_0^2 2x - x^2 \, dx = c \left[x^2 - \frac{x^3}{3} \right]_0^2$$

$$= c\left(4 - \frac{8}{3}\right) = c \, \frac{12 - 8}{3} = c \cdot \frac{4}{3}$$

$$\Rightarrow \quad c = \frac{3}{4}$$

What is $P[X > 1]$? What is $P[X > \frac{1}{2}]$

Calculate $F(x) = \int_{-\infty}^{x} f(y)\,dy$!

$F(x) = 0, \; x \leq 0, \; F(x) = 1, \; x \geq 2$. Otherwise

$$F(x) = \int_0^x \frac{3}{4}(2y - y^2)\,dy = \frac{3}{4}\left[y^2 - \frac{y^3}{3} \right]_0^x$$

$$= 3/4\left(x^3 - \frac{x^3}{3}\right)$$

2.4 Expectation

If throwing a die, we expect that over time the average of points is

$$\frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \frac{1}{6}\cdot 3 + \frac{1}{6}\cdot 4 + \frac{1}{6}\cdot 5 + \frac{1}{6}\cdot 6 = 3.5$$

If tossing a coin n times in a row, we expect that, over many such sequences, the average number of heads is $\frac{n}{2}$.

If tossing a coin until we see the first head, we expect the sequence of tosses to have length

$$\frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 3 + \cdots$$

$$= \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k k = \sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k = 2.$$