Lab 4

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4 Distributions, Densities and Expected Values

4.1 Distribution Function of a Random Variable

Let $\mathcal{X}$ be a random variable with distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \leq x < 1 \\ \frac{3}{5} & 1 \leq x < 2 \\ \frac{7}{8} & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

Compute:

1. $P[\mathcal{X} \leq 1/2]$
2. $P[\mathcal{X} > 1/2]$
3. $P[\mathcal{X} < 3]$
4. $P[\mathcal{X} = 3]$
5. $P[\mathcal{X} < 1]$
6. $P[\mathcal{X} = 1]$

4.2 Sum of Random Variables

Let $\mathcal{X}$ and $\mathcal{Y}$ be two independent random variables, each having the density function

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

Let $\mathcal{Z} = \mathcal{X} + \mathcal{Y}$ be the sum of $\mathcal{X}$ and $\mathcal{Y}$.

1. Find the distribution function of $\mathcal{Z}$.
2. Find the density function of $\mathcal{Z}$.

4.3 Computers Breaking Down

The amount of time, in hours, that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \lambda\, e^{-x/100} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

1. What is the probability that a computer will function between 50 and 150 hours before breaking down?

   Hint: Note that first you have to determine λ so that f is a density.

2. What is the probability that it will function less than 100 hours?

4.4 Expected Value of Exam Ranking

Four men and three women are ranked according to a score in an examination. Suppose that all scores are different, and all rankings are equally likely.

1. Assuming that $\mathcal{X}$ is a random variable denoting the *lowest* ranking achieved by a woman, what is the probability mass function of $\mathcal{X}$?

   Hint: Try to see how Exercise 2.3 can come in handy.

2. Find the expected value $E[\mathcal{X}]$.