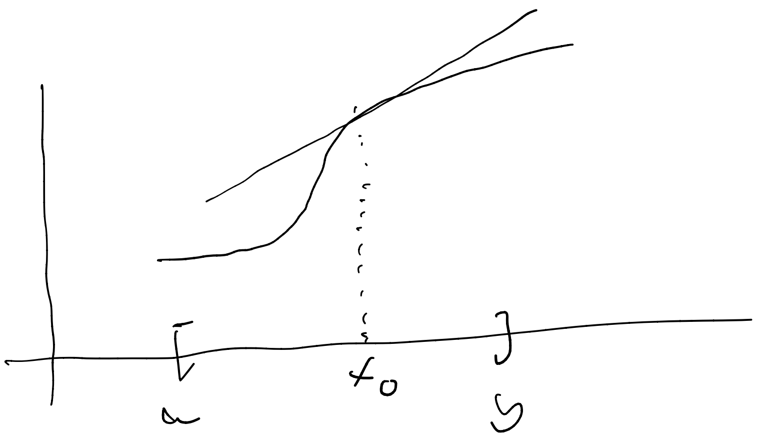
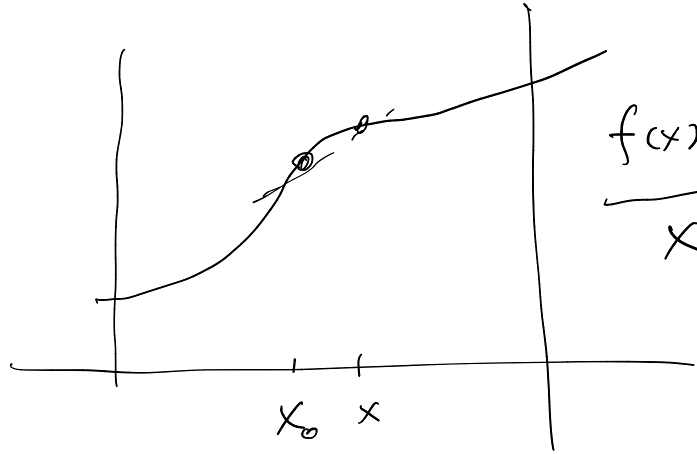
Derivatives and Integrals

$f : [a, b] \longrightarrow \mathbb{R}$





$$\frac{f(x) - f(x_0)}{x_0 - x}$$

Steepness of f in x_0

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$$f(x) = 1 \quad \Rightarrow \quad f'(x_0) = 0 \quad , \text{ f.a. } x_0$$

$$f(x) = x \qquad \lim_{x \to x_0} \frac{x_0 - x}{x_0 - x} = \lim_{\ldots} 1 = 1$$

$$f(x) = g(x)\, h(x)$$

$$\lim_{\ldots} \frac{g(x_0)\, h(x_0) - g(x)\, h(x)}{x_0 - x}$$

$$\lim_{x \to x_0} \frac{g(x_0)\, h(x_0) - g(x)\, h(x_0) + g(x)\, h(x_0) - g(x)\, h(x)}{x_0 - x}$$

$$= \lim_{x \to x_0} \underbrace{\frac{(g(x_0) - g(x))\, h(x_0)}{x_0 - x}}_{g'(x_0)\, \cdot\, h(x_0)} + \underbrace{g(x)}_{g(x_0)} \underbrace{\frac{h(x_0) - h(x)}{x_0 - x}}_{h'(x_0)}$$

$$\Rightarrow \quad f' = g' \cdot h + g \cdot h'$$

$$f(x) = x^2 \Rightarrow f(x) = g(x) \cdot h(x) \,, \quad g(x) = h(x) = x$$

derivative wrt x $\rightsquigarrow$ $\dfrac{d}{dx} x^2 = 2x$

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

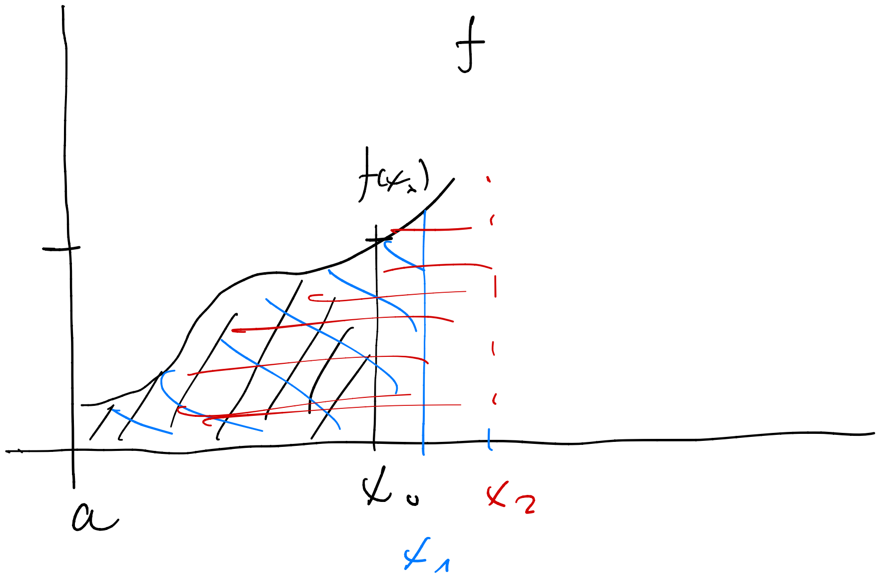
$$= 1 \cdot x + x \cdot 1 \qquad = x + x = 2x$$

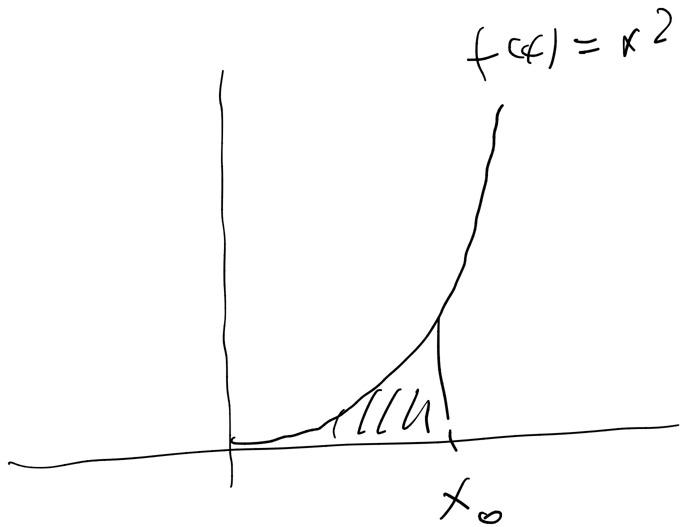
Generally:

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$(f+g)' = f' + g' \,. \qquad (af)' = a \cdot f'$$

$\frac{A}{w} \approx h$





$F(x_0) :=$ "area under the curve of f"

$$\int_a^{x_0} f(y)\,dy$$

Area from x_0 to x_1 : $\int_{x_0}^{x_1} f(y)\,dy$

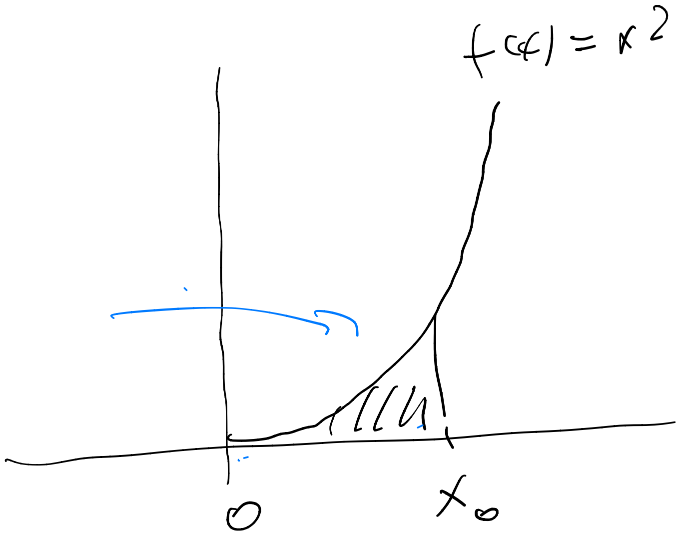
$= F(x_1) - F(x_0)$

Steepness

$$\frac{F(x_1) - F(x_0)}{x_1 - x_0} \approx \frac{\text{Area from } x_0 \text{ to } x_1}{\text{width of area}}$$

$$\lim_{x \to x_0} \frac{F(x) - F(x_0)}{x - x_0} = f(x_0)$$

$$F' = f$$



$$F(x) = \int_0^{x_0} f(x)\,dx$$

Now we know

$$F'(x) = f(x)$$

x^2

with any F

$$\int_0^{x_0} f(x)\,dx$$

$$= F(x_0) - F(0)$$

$$= \frac{1}{3} x_0^3 - \frac{1}{3} 0^3 = \frac{1}{3} x_0^3$$

find a fct F

s.th. $F' = f$ $\left(f(x) = x^2\right)$

find F!

$$\frac{d}{dx} x^3 = 3x^2$$

$$\frac{d}{dx} \boxed{\frac{1}{3} x^3} = \frac{1}{3} \frac{d}{dx} x^3$$

$$= \frac{1}{3}\, 3 \cdot x^2 = \boxed{x^2}$$