2.1 Types of Random Variables

Let $\mathcal{X}$ be discrete

$$p : \mathbb{R} \longrightarrow [0,1]$$

$$p(x) = P[\mathcal{X} = x]$$

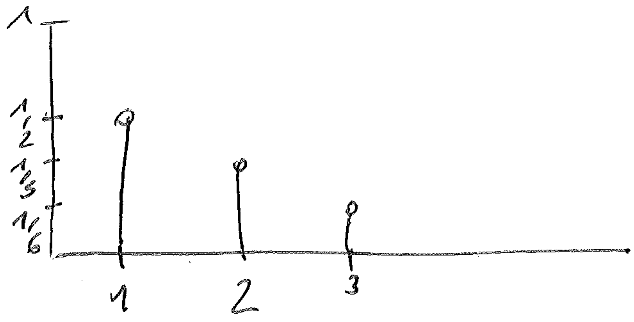
is the probability mass function of $\mathcal{X}$.

Let $x_1, \dots, x_n, \dots$ be the values of $\mathcal{X}$

$$\Rightarrow \sum_{i=1}^{\infty} p(x_i) = 1$$

Example 26: Suppose $\mathcal{X}$ takes values $\{1,2,3\}$, and $p(2) = \frac{1}{3}$, $p(3) = \frac{1}{6}$ $\Rightarrow$ $p(1) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$

Picture



$\square$

Cumulative distribution function:

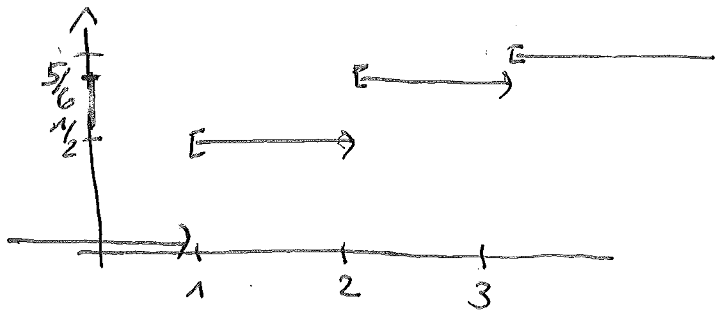
$$F(x) = P[\mathcal{X} \leq x] = \sum_{y \leq x} p(y)$$

Let $x_1, \dots, x_n, \dots$ be the values of a discrete R.V. $\mathcal{X}$, $x_1 < x_2 < x_2 < \dots < x_n < \dots$

$\Rightarrow$ F is constant on each interval $[x_i, x_{i+1})$

$\Rightarrow$ F is a step function

For Ex 26



Definition 27: X is continuous if there exists $f: \mathbb{R} \to \mathbb{R}$, $f \geq 0$, s.th.

$$P[X \in B] = \int_B f(x)\, dx,$$

for all "reasonable" $B \subseteq \mathbb{R}$.

f is the probability density function of X

Idea: Compute the probability of X having values in B by integrating f

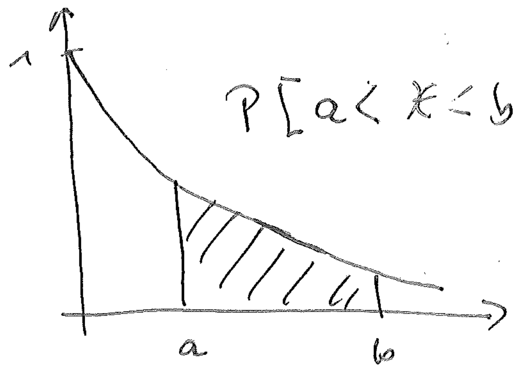
Remark:

$$1 = P[X \in (-\infty, \infty)] = \int_{-\infty}^{\infty} f(x)\, dx$$

$$P[a \leq X \leq b] = \int_a^b f(x)\, dx$$

Example:

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

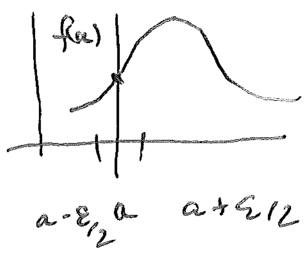


Connection F and f:

$$F(y) = \int_{-\infty}^{y} f(x)\, dx$$

$$\Leftrightarrow \quad F' = f \quad \Leftrightarrow \quad \frac{d}{dy} F(y) = f(y)$$

f continuous



$$P\left[a - \tfrac{\varepsilon}{2} < X < a + \tfrac{\varepsilon}{2}\right] \approx \varepsilon f(a)$$

i.e., probability of a is described by f(a)