Caution: $\mathcal{E}$ ind of $\mathcal{F}, \mathcal{G} \not\Rightarrow \mathcal{E}$ ind of $\mathcal{F} \cap \mathcal{G}$

<u>Example 21</u>: Throw two fair dice.

$\mathcal{E} =$ "$D_1 + D_2 = 7$" $\qquad \mathcal{F} =$ "$D_1 = 1$"

$\qquad \mathcal{G} =$ "$D_2 = 6$"

$$P(\mathcal{E}) = \frac{1}{6} \qquad P(\mathcal{F}) = \frac{1}{6} \qquad P(\mathcal{G}) = \frac{1}{6}$$

$$P(\mathcal{E}\mathcal{F}) = \frac{1}{36} \qquad P(\mathcal{E}\mathcal{G}) = \frac{1}{36}$$

$$P(\mathcal{E}\mathcal{F}\mathcal{G}) = \frac{1}{36}, \quad \text{but } P(\mathcal{E})P(\mathcal{F}\mathcal{G}) = \frac{1}{6} \cdot \frac{1}{36}$$

<u>Definition 22</u>: <u>$\mathcal{E}, \mathcal{F}, \mathcal{G}$</u> are <u>independent</u> if

- $\mathcal{E}, \mathcal{F}$ and $\mathcal{E}\mathcal{G}$ and $\mathcal{F}\mathcal{G}$ are indepent
- $P(\mathcal{E}\mathcal{F}\mathcal{G}) = P(\mathcal{E})P(\mathcal{F})P(\mathcal{G})$

Remark: $\mathcal{E}, \mathcal{F}, \mathcal{G}$ independent $\Rightarrow \mathcal{E}$ and $\mathcal{F} \cup \mathcal{G}$ independent

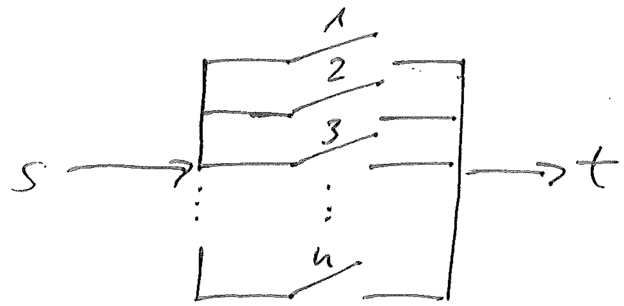
$$P(\mathcal{E}(\mathcal{F} \cup \mathcal{G})) = P(\mathcal{E}\mathcal{F} \cup \mathcal{E}\mathcal{G})$$
$$= P(\mathcal{E}\mathcal{F}) + P(\mathcal{E}\mathcal{G}) - P(\mathcal{E}\mathcal{F}\mathcal{G})$$
$$= P(\mathcal{E})P(\mathcal{F}) + P(\mathcal{E})P(\mathcal{G}) - P(\mathcal{E})P(\mathcal{F})P(\mathcal{G})$$
$$= P(\mathcal{E})\,(P(\mathcal{F}) + P(\mathcal{G}) - P(\mathcal{F}\mathcal{G}))$$
$$= P(\mathcal{E}) \cdot P(\mathcal{F} \cup \mathcal{G})$$

Generalize: $\mathcal{E}_1, \dots, \mathcal{E}_n$ independent iff for every subset $\mathcal{F}_1, \dots, \mathcal{F}_m$:

$$P(\mathcal{F}_1 \dots \mathcal{F}_m) = P(\mathcal{F}_1) \dots P(\mathcal{F}_m)$$

Example: sequences of experiments (rolling many dice)

<u>Example 23</u>



Components are independent, work with prob. p_i

$$P(\text{system works}) = ?$$

$\mathcal{E}$ = "system works", $\mathcal{F}_i$ = "component i works"

$$P(\mathcal{E}) = 1 - P(\overline{\mathcal{E}}) = 1 - P(\overline{\mathcal{F}_1} \cdots \overline{\mathcal{F}_n})$$

$$= 1 - \prod_{i=1}^{n} P(\overline{\mathcal{F}_i}) = 1 - \prod_{i=1}^{n} (1 - P(\mathcal{F}_i))$$

$$= 1 - \prod_{i=1}^{n} (1 - p_i)$$