2.1 Types of Random Variables

Let X be discrete.

$$p : \mathbb{R} \longrightarrow [0,1]$$

$$p(x) = P[X = x]$$ (pmf)

is the probability mass function of X

Let $x_1, \ldots, x_n, \ldots$ be the possible values of X

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

Example 26: Let X takes values $\{1, 2, 3\}$

and $p(2) = \frac{1}{3}$, $p(3) = \frac{1}{6} \Rightarrow p(1) = \frac{3}{6} = \frac{1}{2}$

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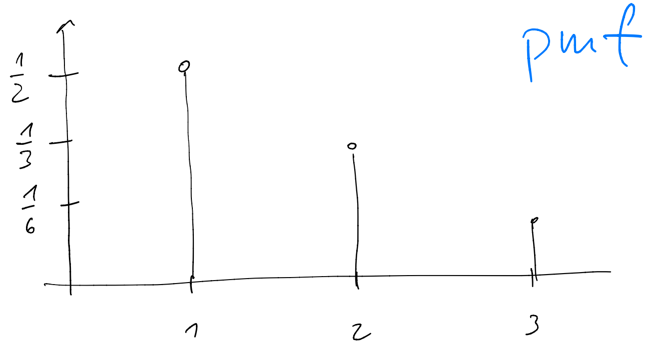
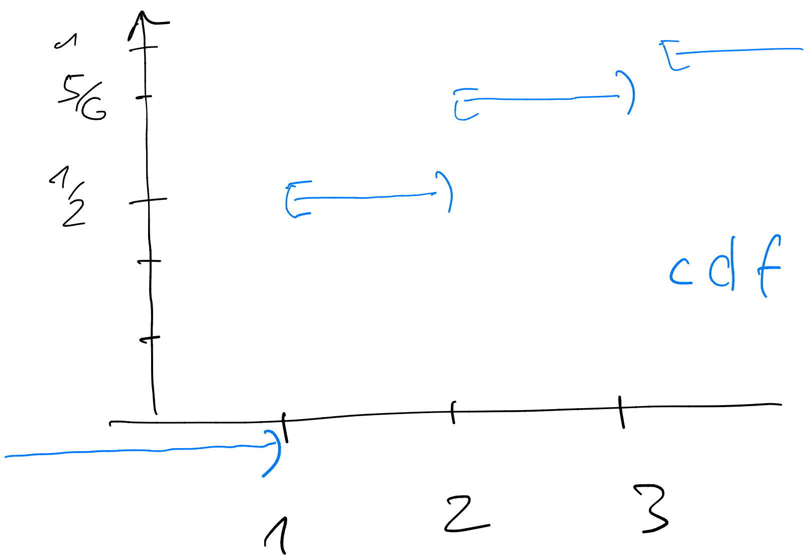


Cumulative distribution function

$$F(x) = P[X \leq x] = \sum_{y \leq x} p(y)$$

In general:

- F is constant on each interval $[x_i, x_{i+1})$
- F is a step function



Definition 27: X is **continuous** if there is a fct

$f : \mathbb{R} \to \mathbb{R}$, $f \geq 0$, sth (= such that)

$$P[X \in B] = \int_B f(x)\,dx$$

for all "reasonable" $B \subseteq \mathbb{R}$. We call f the

$\underbrace{\text{reasonable}}_{\text{essentially unions intervals}}$

probability density function of X (pdf)

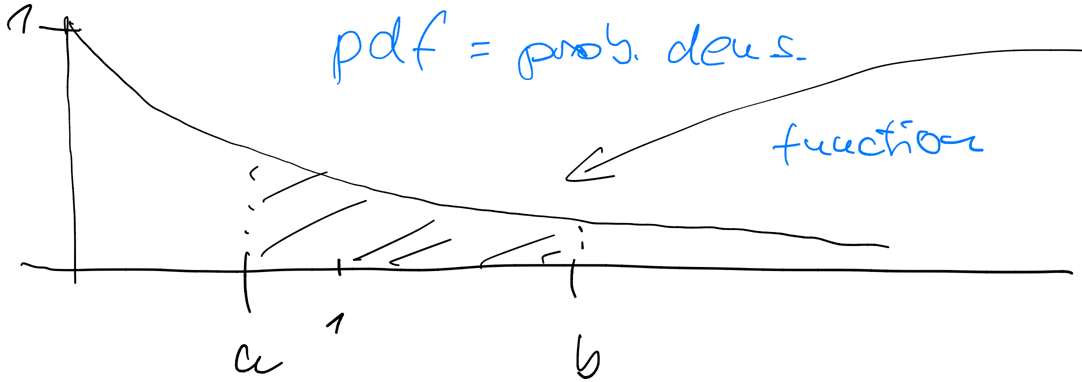
Remark:

$$1 = P[X \in (-\infty, \infty)] = \int_{-\infty}^{\infty} f(x)\,dx$$

$$P[a \leq X \leq b] = \int_a^b f(x)\,dx$$

Example:

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad F(x) = \{$$



pdf = prob. dens. function

$P[a < X < b]$

= area under the graph of f between a and b

For a discrete X we would take

$$\sum_{a<x<b} p(x)$$

Connection between cdf and pdf here:

$$F(x) = P[X \leq x] = \int_{-\infty}^{x} f(y)\,dy$$

$$\Longleftrightarrow F'(x) = f(x) \quad \left(\wedge \lim_{x \to -\infty} F(x) = 0\right)$$

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What is F?

$$f(y) = \begin{cases} e^{-y} & y \ge 0 \\ 0 & y < 0 \end{cases}$$

$$\boxed{F(x) = \int_{-\infty}^{x} f(y)\,dy} = \begin{cases} \int_{-\infty}^{x} 0\,dy = \boxed{0} & \text{if } x < 0 \\ \int_{-\infty}^{x} f(y)\,dy & \text{if } x \ge 0 \end{cases}$$

$$\int_{-\infty}^{x} f(y)\,dy = \int_{-\infty}^{0} f(y)\,dy + \int_{0}^{x} f(y)\,dy = 0 + \int_{0}^{x} e^{-y}\,dy$$

$$= \left[-e^{-y}\right]_0^x = -e^{-x} - (-e^{-0}) = -e^{-x} + 1 = \boxed{1 - e^{-x}}$$

This is the cdf of the exp. distribution