2 Random Variables

We roll 2 dice: the sum $X = D_1 + D_2$,

$$X : \Omega \longrightarrow \mathbb{R}$$

is a random variable

Idea: We are not interested in arbitrary events but only events that can be described by X having certain values

E.g. weight ≥ 100 kg, height < 1.60 m, ---.

Back to dice

$$P([X = 2]) = \frac{1}{36}$$

$$P([X = 3]) = \frac{2}{36}$$

$$P([X = 4]) = \frac{3}{36}$$

$$P([X = 5]) = \frac{4}{36}$$

$$P([X = 6]) = \frac{5}{36}$$

$$P([X = 7]) = \frac{6}{36}$$

$$P([X = 8]) = \frac{5}{36}$$

$$P([X = 9]) = \frac{4}{36}$$

$$P([X = 10]) = \frac{3}{36}$$

$$P([X = 11]) = \frac{2}{36}$$

$$P([X = 12]) = \frac{1}{36}$$

check

$$\sum_{i=2}^{12} P([X = i]) = 1$$

Other pass. events expressible by X:

$$P[5 \leq X \leq 9] = \frac{24}{36}$$

A random variable $X : \Omega \to \mathbb{R}$

is <u>discrete</u> if it has only finitely (or countably) many values $x_1, \ldots, x_n, \ldots$

X is <u>continuous</u> if it takes a continuum of values (e.g. weight, ...)

<u>Definition:</u> The <u>(cumulative)</u> distribution function of X is

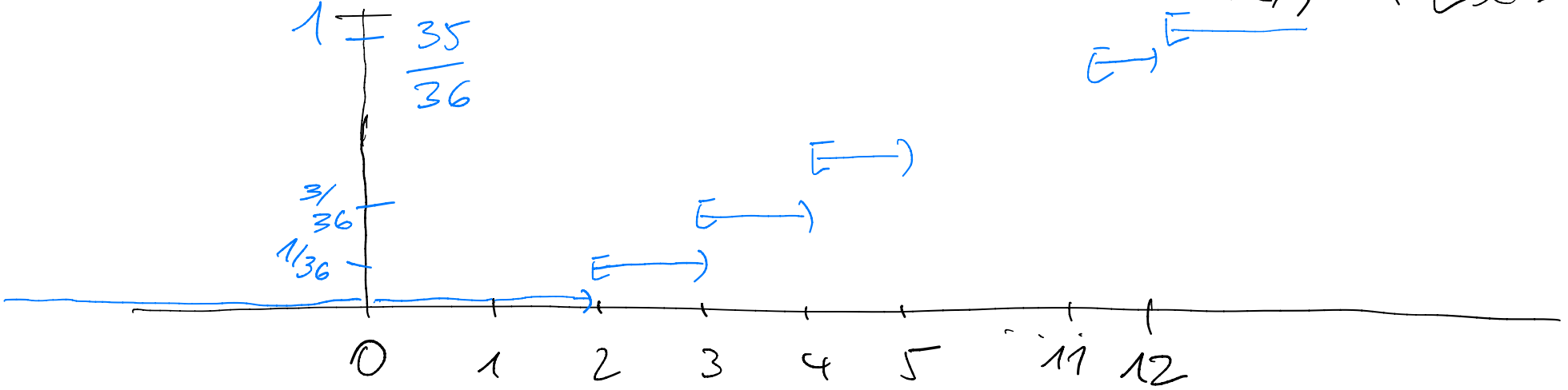
$$F : \mathbb{R} \to [0,1]$$

$$F(x) = P[X \leq x]$$

"$X \sim F$" means "F is distribution of X"

Distribution for $X = D_1 + D_2$

$F(x) = P[X \leq x]$



$F(2) = P[X \leq 2]$

$F(3) = P[X \leq 3] = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$

F answers all probability questions about X:

Eg $\quad P[a < X \leq b] = ?$

$$[X \leq b] = [X \leq a] + [a < X \leq b]$$

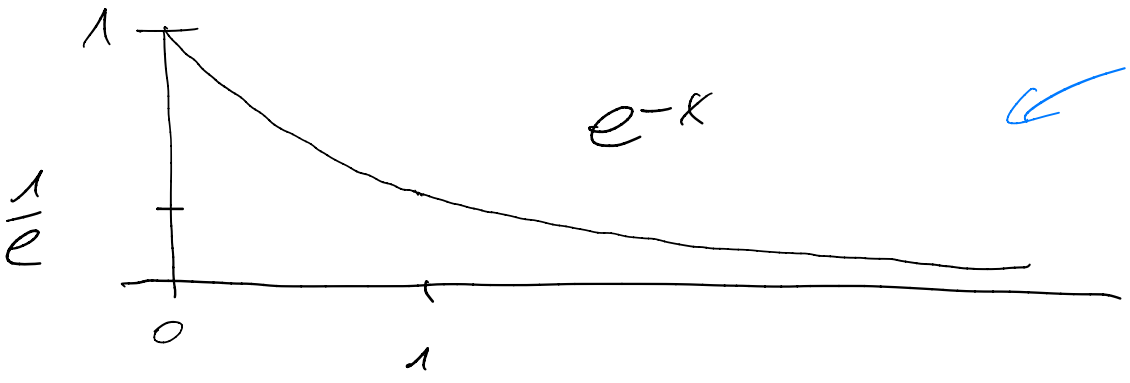
$$P[a < X \leq b] = P[X \leq b] - P[X \leq a]$$

$$= F(b) - F(a)$$

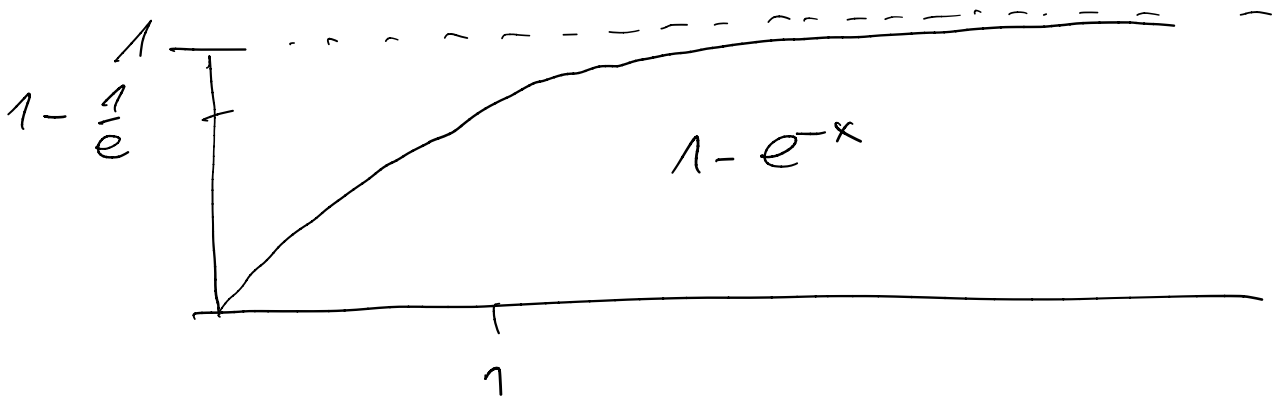
Example 25 a) Suppose $X \sim F$

e.g., time until a device breaks, an atom decays

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 1 - e^{-x}, & x > 0 \end{cases}$$



density of the exponential distribution

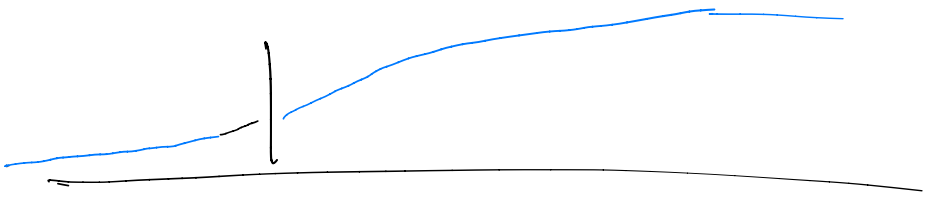


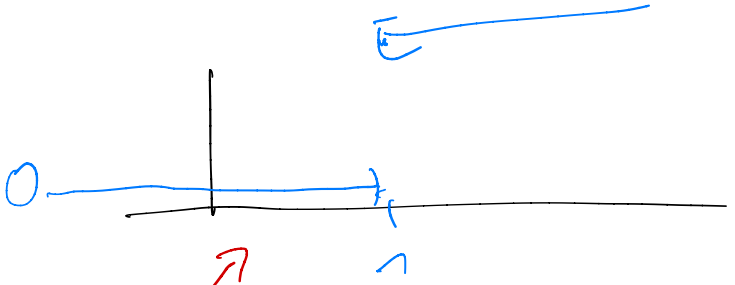
$F(x) = P[X \leq x]$

All distribution functions satisfy

- $0 \leq F(x) \leq 1$ (since $F(x) = P[X \leq x]$ is a probability)
- F is monotonically increasing
- $\lim_{x \to -\infty} F(x) = 0$
- $\lim_{x \to +\infty} F(x) = 1$

Possible shapes





X is discrete and the only possible value is 1