Lab 2 **Pietro Galliani, Werner Nutt**
Andy Rivkin

2 Independence and Uniform Probabilities

2.1 Independent Dice Throws

A pair of fair dice is rolled. Let D_1 denote the number on the first die and D_2 the one on the second. Consider the following events:

$$\begin{aligned}
\mathcal{E} &= \text{“}D_1 + D_2 = 4\text{”} \\
\mathcal{F} &= \text{“}D_1 + D_2 = 5\text{”} \\
\mathcal{G} &= \text{“}D_1 \text{ is even”} \\
\mathcal{H} &= \text{“}D_1 + D_2 \text{ is even”}
\end{aligned}$$

For each combination of two events, find out whether they are independent or not.

2.2 Birthdays

We want to investigate the following question:

> Consider a set of 23 unrelated people. What is the probability that at least two of them have the same birthday?

Here are some hints how to do that:

- Consider the dual problem: what is the probability that no two people share the same birthday?
- Solve it first for 2 and 3 people. Then generalize to an arbitrary number n.
- Develop a formula that expresses the probability that among n people no two people have the same birthday. For which values of n is your formula correct?
- How would you compute that probability?

2.3 Exam Rankings

Four men and three women are ranked according to a score in an examination. Suppose that all scores are different, and all rankings are equally likely. The highest ranking is 1, the lowest 7. If $\mathcal{X}$ denotes the *lowest* ranking achieved by a woman, find $P[\mathcal{X} = i], 1 \leq i \leq 7$.