Frequentist view: n experiments

$F \approx n \cdot P(F)$ many times

$EF \approx n \cdot P(EF)$ many times

Among $n \cdot P(F)$ many F-outcomes, there are $n \cdot P(EF)$ many EF-outcomes

$$\Rightarrow \quad P(E \mid F) = \frac{n \cdot P(EF)}{n \cdot P(F)} = \frac{P(EF)}{P(F)}$$

<u>Example 12</u> Box with 32 transistors:
20 <u>working</u>, 8 <u>partly working</u>, 4 <u>defective</u>.
Choose one. If it does not fail, what is the probability that it is working?

$$P(W \mid \bar{D}) = \frac{P(W\bar{D})}{P(\bar{D})} = \frac{P(W)}{P(W \cup P)}$$

$$= \frac{\frac{20}{32}}{\frac{28}{32}} = \frac{20}{28} = \frac{5}{7} \qquad \square$$

<u>Example 13</u> Toss a coin twice. What is

$$P(2 \text{ heads} \mid \geq 1 \text{ head}) = ?$$

$$P(\cdot) = \frac{P(2 \text{heads} \cap \geq 1 \text{ head})}{P(\geq 1 \text{ head})} = \frac{P(\{(h,h)\})}{P(\{(t,h),(h,h),(h,t)\})}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

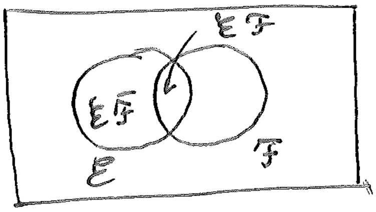
Example 14: A football team has 20% chance of reaching the final. If in the final, it has 50% winning chance.

$$P(\text{Champion}) = ?$$

$$P(\text{Champion}) = P(FW) = P(W \mid F)\, P(F) = 0.5 \cdot 0.2 = 0.1$$

1.5 Bayes' Formula

Note: E, F events



$$E = EF \cup E\bar{F}$$

$\Rightarrow$

$$P(E) = P(EF) + P(E\bar{F})$$

$$= P(E|F)P(F) + P(E|\bar{F})P(\bar{F})$$

$\Rightarrow$ $P(E)$ can be computed by conditioning on some F

Example 15 People are risk takers (30%) or not. Every year, 40% of risktakers have an accident, and 20% of non-risk-takers. What is $P(A)$?

$$P(A) = P(A|R)P(R) + P(A|\bar{R})P(\bar{R})$$

$$= 0.4 \cdot 0.3 + 0.2 \cdot 0.7 = 0.12 + 0.14 = 0.26$$

Updating beliefs in the presence of new information

Example 16 A random person has an accident. What is the probability, this is a risk taker?

$$P(R|A) = \frac{P(RA)}{P(A)} = \frac{P(A|R)P(R)}{P(A)} = \frac{0.4 \cdot 0.3}{0.26} = \frac{0.12}{0.26}$$

$$= \frac{6}{13} = 0.46\ldots$$

Recall

$$P(A|R) = \frac{P(AR)}{P(R)} \Rightarrow P(AR) = P(A|R)P(R)$$

Bayes' Formula, simple version

$$P(F \mid E) = \frac{P(E \mid F)\, P(F)}{P(E)}$$

Example A new test for the early diagnosis of cancer has been developed with the following characteristics:

Events: T test positive C person has cancer

$$P(T|C) = \frac{99}{100} \qquad P(T|\bar{C}) = \frac{1}{100}$$

$$P(C) = \frac{1}{100}$$

What is $P(C|T)$?

$$P(C|T) = \frac{P(T|C)\,P(C)}{P(T)}$$

$$= \frac{P(T|C)\,P(C)}{P(T|C)P(C) + P(T|\bar{C})P(\bar{C})}$$

$$= \frac{\frac{99\cdot 1}{100\cdot 100}}{\frac{99\cdot 1 \;+\; 1\cdot 99}{100\cdot 100}}$$

$$= \frac{1}{2}$$

That is: In 50% of all cases the test result is wrong

Note: To judge the quality of a test, we need to know not only $P(T|C)$, but also $P(T|\bar{C})$ (= false positives) and $P(C)$