Subjective (= Bayesian) view of probabilities

We have seen the subj. view of cond. prob.

Frequentist view: n experiments

$\mathcal{F} \simeq n \cdot P(\mathcal{F})$ many times

$\mathcal{E}\mathcal{F} \simeq n \cdot P(\mathcal{E}\mathcal{F})$ many times

Ignore outcomes not $\mathcal{F}$.

Among the $n \cdot P(\mathcal{F})$ many $\mathcal{F}$-outcomes, there are $n\, P(\mathcal{E}\mathcal{F})$ many $\mathcal{E}\mathcal{F}$-outcomes

$$\Rightarrow P(\mathcal{E} \mid \mathcal{F}) = \frac{n \cdot P(\mathcal{E}\mathcal{F})}{n \cdot P(\mathcal{F})} = \frac{P(\mathcal{E}\mathcal{F})}{P(\mathcal{F})}$$

Example 12   Box with 32 transistors:

20 working, 8 partly working, 4 deficient

Exp: Choose 1 transistor,

Suppose it does not fail. What is the prob. that it is working?

Three events: W, P, D (pick a working, ... transistor)

$$P(W \mid \bar{D}) = \frac{P(W\bar{D})}{P(\bar{D})} = \frac{P(W)}{P(W \cup P)} = \frac{\frac{20}{32}}{\frac{28}{32}} = \frac{20}{28} = \frac{5}{7}$$

$$P(E \mid F) = \frac{P(EF)}{P(F)}$$

Quiz 7: Tossing Coins

$$P(\underbrace{2 \text{ heads}}_{\mathcal{E}} \mid \underbrace{\geq 1 \text{ head}}_{\mathcal{F}}) =$$

$$= \frac{P(\mathcal{E}\mathcal{F})}{P(\mathcal{F})} = \frac{P(2 \text{ heads} \wedge \geq 1 \text{ head})}{P(\geq 1 \text{ head})}$$

$$= \frac{P(\{(H,H)\})}{P(\{(H,H),(T,H),(H,T)\})} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

$$\mathcal{S} = \{(H,H), (H,T), (T,H), (T,T)\}$$

$\mathcal{E}$

$\mathcal{F}$

Quiz 8: Champions

reaching the final

winning the final

$$P(\text{Champions}) = P(\mathcal{F}\,W) = P(W|\mathcal{F})\,P(\mathcal{F})$$

$$= .5 \times .2 = \underline{\underline{0.1}}$$

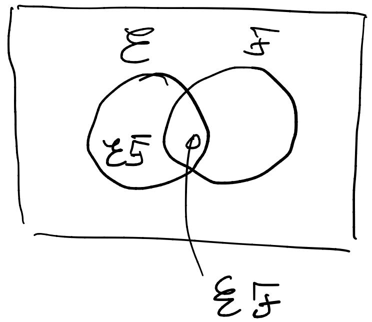
$$P(\mathcal{F}) = .2$$

$$P(W|\mathcal{F}) = .5$$

$$P(W|\mathcal{F}) = \frac{P(W\mathcal{F})}{P(\mathcal{F})}$$

$$\Rightarrow P(W|\mathcal{F})\,P(\mathcal{F}) = P(W\mathcal{F})$$

1.5 Bayes' Formula



E, F events $\Rightarrow$ $E = EF \,\dot{\cup}\, E\bar{F}$

$$\Rightarrow P(E) = P(EF) + P(E\bar{F})$$

$$= P(E|F)\,P(F) + P(E|\bar{F})\,P(\bar{F})$$

Here: $P(E)$ can be computed by conditioning on some F

special case of the law of total probability (LOTP)

Example 15 Insurance company: People are risk taker (R) (30%) or not. Every year, 40% of risk takers have an accident (A), only 20% of non-risk takers. What is $P(A)$?

$$P(A) = P(A|R)\,P(R) + P(A|\bar{R})\,P(\bar{R})$$

$$= .4 \times .3 + .2 \times .7 = .12 + .14 = 0.26$$

Updating beliefs in the presence of new information

Example 16: Suppose, a client has an accident.

What is the prob. this was a risk taker?

$$\boxed{P(R \mid A)} = \frac{P(RA)}{P(A)} = \boxed{\frac{P(A \mid R)\,P(R)}{P(A)}}$$

Bayes' Law

Remember:

$$P(A \mid R) = \frac{P(AR)}{P(R)} = \frac{P(RA)}{P(R)}$$

$$\Rightarrow P(RA) = P(A \mid R)\,P(R)$$

$$= \frac{.4 \times .3}{0.26}$$

$$= \frac{.12}{.26} = \frac{12}{26} = \frac{6}{13}$$

$$= 0.46$$

Bayes' Formula

$$P(\mathcal{F} \mid \mathcal{E}) = \frac{P(\mathcal{E} \mid \mathcal{F})\, P(\mathcal{F})}{P(\mathcal{E})}$$

Quiz 9: Testing for a Disease

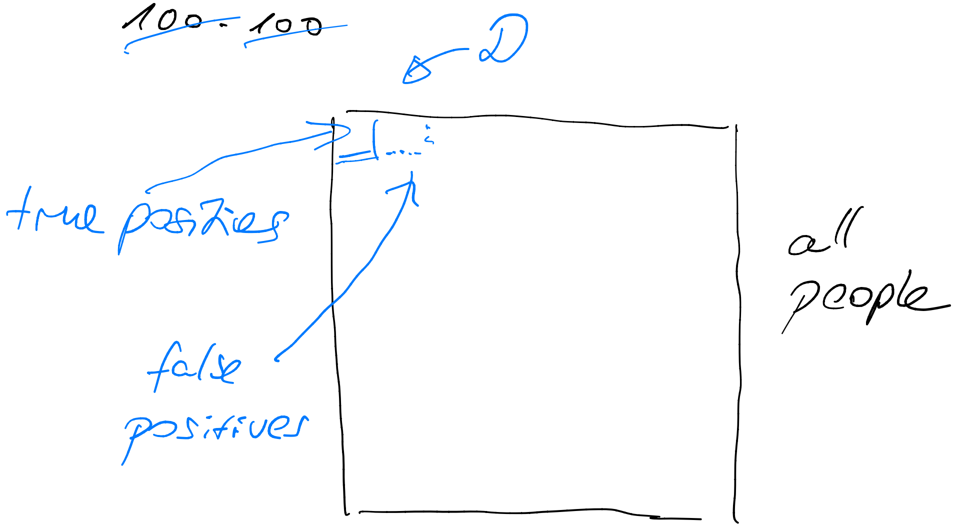
$\mathcal{D}$ person has disease $\qquad\qquad$ $\mathcal{T}$ test is positive

$$P(\mathcal{T} \mid \mathcal{D}) = \frac{99}{100} \qquad P(\mathcal{T} \mid \bar{\mathcal{D}}) = \frac{1}{100} \qquad P(\mathcal{D}) = \frac{1}{100}$$

$$P(\mathcal{D} \mid \mathcal{T}) = \frac{P(\mathcal{T} \mid \mathcal{D})\, P(\mathcal{D})}{P(\mathcal{T})} = \frac{\dfrac{\cancel{99} \cdot 1}{\cancel{100} \cdot \cancel{100}}}{\dfrac{2 \cdot \cancel{99}}{\cancel{100 \cdot 100}}} = \frac{1}{2}$$

$$P(\mathcal{T}) = P(\mathcal{T} \mid \mathcal{D})\, P(\mathcal{D}) + P(\mathcal{T} \mid \bar{\mathcal{D}})\, P(\bar{\mathcal{D}})$$

$$= \frac{99}{100} \cdot \frac{1}{100} + \frac{1}{100} \cdot \frac{99}{100}$$



LOTP