1   Probabilities

1.1 Counting Heads

An experiment consists of throwing a fair coin four times.

1. Give a formal description of the sample space.

    Hint: What is does a possible outcome of this experiment? How can you model it mathematically?

2. For each $k = 0, 1, \ldots, 4$, describe the event

    $$\mathcal{E}_k = \text{"The number of heads thrown is } k\text{"}$$

    as a subset of the sample space.

3. How many outcomes does $\mathcal{E}_k$ comprise? Assuming equal probability of each outcome (fair coin!), what are the probabilities of $\mathcal{E}_0, \ldots, \mathcal{E}_4$?

1.2 Balls in a Line

Three red balls, four green balls, and four blue balls are lined up in a random order (that is, each ordering is equally likely).

1. How many possible ways are there to order these balls, if we can distinguish all balls? (Assume for instance that each ball carries a number between 1 and 11.)

2. What is the probability that in a random line-up all the balls of each color are together, that is, first there are all the balls of one colour, then the ones of another colour, and finally all the ones of the third colour?

3. What is the probability that the last three balls are red?

4. Knowing that the third ball is green, what is the probability that the last two balls are red?

5. Knowing that the ninth ball is blue, what is the probability that the last three balls are blue?

6. Knowing that the ninth ball is green, what is the probability that the last three balls are blue?

1.3 What Kind of Taxi Was Involved in the Accident?

A taxi was involved in a hit and run accident at night. Two taxi companies, the Green and the Blue, operate in the city. You are given the following data:

- 85% of the taxis in the city are Green and 15% are Blue.

- A witness identified the taxi as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colours 80% of the time and failed 20% of the time.

What is the probability that the taxi involved in the accident was Blue rather than Green?