Probability Theory and Statistics
Exam

June 2019

Name: ______________________________

ID: ______________________________

Question	1	2	3	4	5	6	Total
Points	7	8	6	6	8	(+2)	35
Reached							

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Question 1. 7 P.

Three red balls, four green balls, and four blue balls are lined up in a random order (that is, each ordering is equally likely).

1. How many possible orderings exist?
2. What is the probability that all the balls of each color are together?
3. Knowing that the third ball is green, what is the probability that the last two balls are red?
4. Knowing that the ninth ball is green, what is the probability that the last three balls are blue?
5. If $\mathcal{X}$ denotes the earliest position of a ball that is *not* blue, find $E[\mathcal{X}]$

Hint: it suffices to give your answer as numerical expressions, like e.g., $\frac{7\cdot3+5}{30}$ (this is **not** the answer), but try to simplify as much as possible.

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Question 2. 8 P.

The joint probability density function of $\mathcal{X}$ and $\mathcal{Y}$ is

$$f(x, y) = c \cdot \left(3y^2 + 2xy\right) \qquad 0 < x < 1; \quad 0 < y < 1$$

1. Find the value c
2. Compute the density function of $\mathcal{X}$
3. Find $P[\mathcal{X} + 2\mathcal{Y} \leq 1]$
4. If $E[\mathcal{X}] = \mu$, approximate $P[\mathcal{X} \leq 0]$

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Question 3. 6 P.

We throw a fair die repeatedly. If the die falls on 1,2, or 3, we say that it is a *low throw* (otherwise, it is a *high throw*). Approximate the probability of:

1. seeing at most 55 low throws, in 100 throws of the coin,
2. seeing at least 215 heads, in 400 throws.

Suppose now that you win 1 euro for each low throw, and lose 1 euro for each high throw.

3. what is the probability of winning more than 10 euros after 100 throws?
4. how many throws do you need to guarantee that the probability of winning at least 10 euros is greater than 0.99?

Justify your answers.

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Question 4. 6 P.

The weight of 16 randomly chosen containers is measured. The observed sample mean is 715kg and the sample standard deviation is 8kg.

1. Find a value c such that, with probability 99%, the difference between the observed mean and the true mean is less than c.
2. Compute the p-value for the hypothesis $H_0 : \mu \geq 721$. Approximate it as well as you can from the probability tables provided.
3. What would you need to change in your calculations if we knew that the *population* standard deviation is 8kg?

Justify your answers.

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Question 5. 8 P.

Let $\mathcal{X}$ be a random variable with the density function

$$f(x) = \begin{cases} 1-\theta+x & \text{if } \theta-1 \le x \le \theta \\ 1+\theta-x & \text{if } \theta < x \le \theta+1 \\ 0 & \text{otherwise.} \end{cases}$$

where θ is an unknown parameter. Suppose that we have a sample $\mathcal{X}_1, \ldots, \mathcal{X}_n$ from this variable.

1. Compute $E[\mathcal{X}]$ and $Var(\mathcal{X})$
2. Determine an *unbiased* point estimator for θ
3. Approximate $100(1-\alpha)\%$ two-sided confidence intervals for θ (assume that n is large enough)
4. If a sample of size 6 yields the values 2, 2.5, 3, 3, 3.5, 4, approximate a 95% one-sided upper confidence interval for θ.

Justify your answers.

Hint: An estimator $\widehat{\theta}$ of the parameter θ is *unbiased* if $E[\widehat{\theta}] = \theta$.

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Question 6. **Bonus** (+2 P.)

Give the name and the density function of one distribution covered in the course.