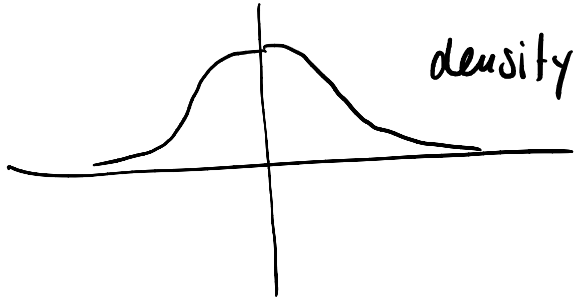
The ==normal distribution== has a density (bell curve)

Parameters μ, σ^2

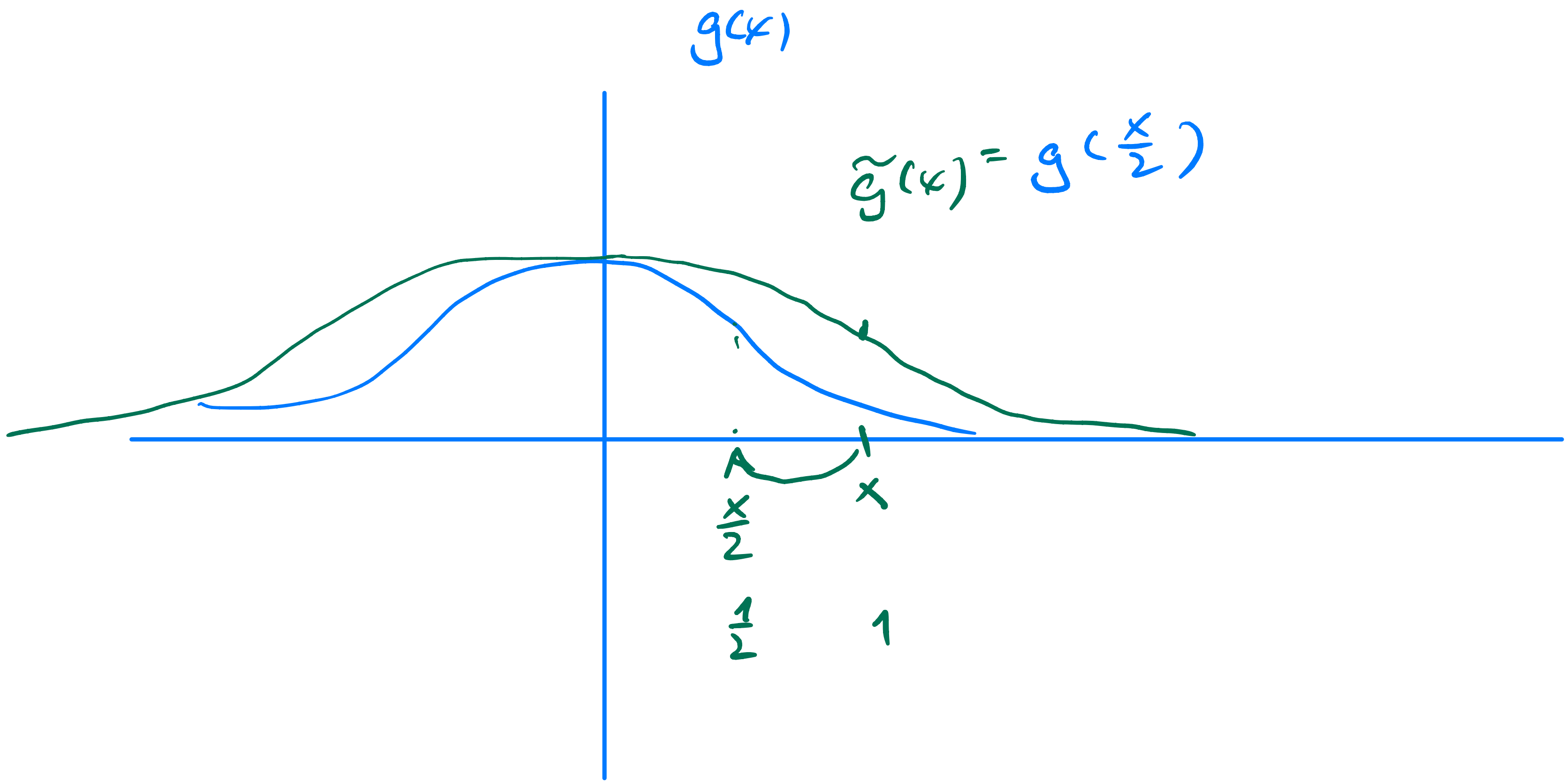


$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$K\, a^{-x^2}$

$a = e^{-\alpha} \Rightarrow a^{x^2} = e^{-\alpha x^2}$

$e^{-\alpha x^2}$ with $\alpha = \frac{1}{2\sigma^2}$



Dividing the argument of a function by a number > 1, e.g. by 2, makes the graph wider.

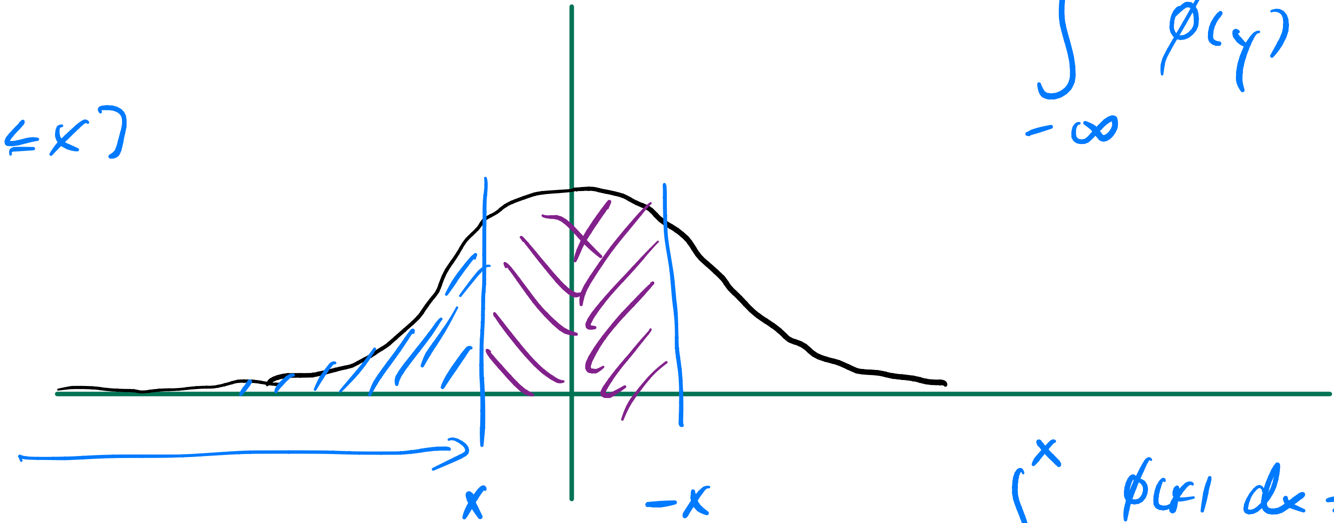
Hence, the bigger σ in the normal density, the wider the graph, the greater the variance of the distribution.

Tabellen der Normalverteilung

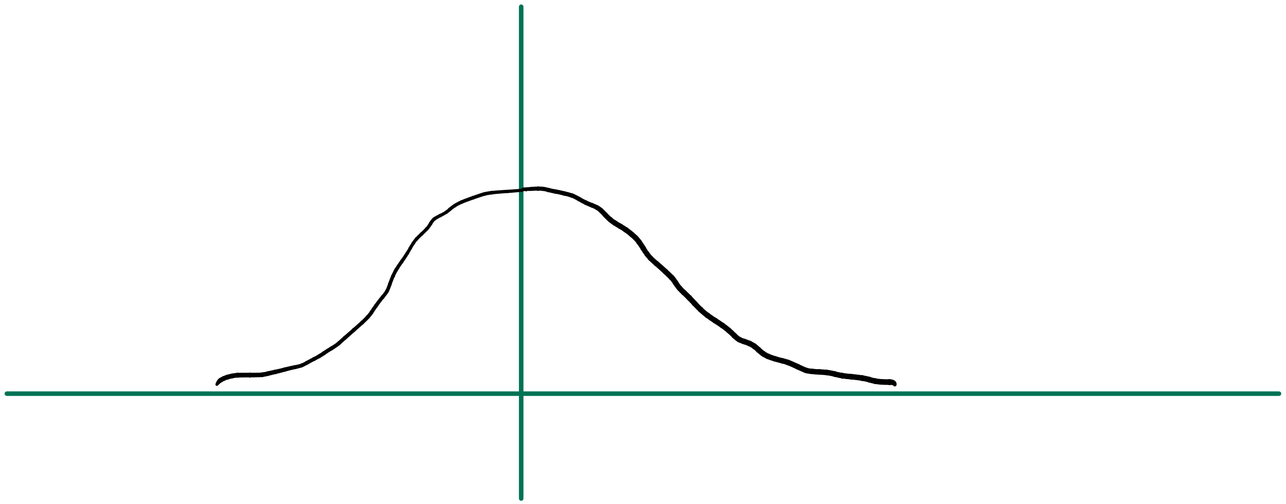
$Z \sim \mathcal{N}(0,1)$

$$\int_{-\infty}^{0} \phi(y)\, dy = \frac{1}{2}$$

$P[Z \leq x]$



$$\int_{-\infty}^{x} \phi(x)\, dx = \frac{1}{2} - \int_{-x}^{0} \phi(y)\, dy$$



$E[X+b] = E[X] + b$, $E[aX] = a \cdot E[X]$ $\quad Var(X+Y) = Var(X) + V(Y)$

Korollar: Seien $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, $a, b \in \mathbb{R}$.
Dann gilt

- $aX + b \sim N(a\mu_X + b, a^2\sigma_X^2)$
- $X + Y \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$

Wir schreiben $N(0,1)$-verteilte ZVen als Z.

Satz: Seien $Z \sim N(0,1)$, $X \sim N(\mu, \sigma^2)$. Dann gilt

- $\sigma Z + \mu \sim N(\mu, \sigma^2)$
- $\frac{X - \mu}{\sigma} \sim N(0,1)$

$X \sim \mathcal{N}(\mu, \sigma^2)$

Turn X into $Z \sim \mathcal{N}(0,1)$

How:

X had mean μ, var. σ^2

Z has mean 0, var 1

1) Turn X into a RV with mean 0

$X - \mu$

2) Turn $X - \mu$ into a RV with Var (= St. dev) = 1

$$\frac{X-\mu}{\sigma} : \operatorname{Var}\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2}\operatorname{Var}(X-\mu)$$

$$= \frac{1}{\sigma^2}\operatorname{Var}(X) = \frac{1}{\sigma}\cdot\sigma^2 = 1$$

$Z \sim \mathcal{N}(0,1)$

$\longrightarrow \quad X \sim \mathcal{N}(\mu, \sigma^2)$

go in reverse direction:

- multiply by σ
- add μ

$\sigma Z + \mu$